line running down the stack is the group of 24 pipes leading to the pressure gauge and the holes are located about 2 feet above the top of the pipes. The stack at this point is 11.8 feet in diameter.

Space does not permit any detailed statement or comparison of the results. A full account of the work is given in *Research Paper* **221** appearing in the September, 1930, issue of the Bureau of Standards *Journal of Research.* The general conclusions drawn from the tests may be stated as follows:

1. The wind pressure on a chimney at a given wind speed is a function of the ratio of the height of the chimney to its diameter and possibly also of the roughness of its surface.
2. Experiments on small cylinders cannot be directly used to predict the wind pressure on a full scale chimney because of the large scale-effect.
3. A wind pressure corresponding to 20 lbs. per square foot of projected area at a wind speed of 100 miles per hour is a safe value to use in designing chimneys of which the exposed height does not exceed 10 times the diameter.
4. The pressure may reach large values locally and this may need consideration in the design of thin-walled stacks of large diameter.
5. Further experiments are necessary to obtain satisfactory information as to the variation of wind pressure with the ratio of height to diameter.

PROJECTIVE NORMAL COÖRDINATES

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A projective geometry of paths was formulated first by T. Y. Thomas,[1] and included the definition of a type of projective normal coördinates. Veblen and J. M. Thomas[2] proposed a system of such coördinates as solutions of a certain system of partial differential equations in n dependent variables. In §41 of my Non-Riemannian Geometry[3] I showed that the determination of these coördinates could be reduced to the solution of a differential equation in one dependent variable. It is the purpose of this note to give this equation another form and thence obtain the explicit form of the expressions of general coördinates in terms of projective normal coördinates.

1. Consider an n-dimensional space V_n of coördinates x^i. We take as the basic elements of V_n the *paths*, that is, the integral curves of the system of differential equations.

$$\frac{dx^i}{dt}\left(\frac{d^2x^j}{dt^2} + \Gamma^j_{kl}\frac{dx^k}{dt}\frac{dx^l}{dt}\right) - \frac{dx^j}{dt}\left(\frac{d^2x^i}{dt^2} + \Gamma^i_{kl}\frac{dx^k}{dt}\frac{dx^l}{dt}\right) = 0, \quad (1.1)$$

where Γ^i_{kl} are functions of the x's, and without loss of generality we take $\Gamma^i_{kl} = \Gamma^i_{lk}$. Throughout the paper we use the convention that when the same index appears as a subscript and superscript in a term, as in the case of k above, this term stands for the sum of terms as the index takes the values 1 to n. The independent variable t in (1.1) is a general one and equations (1.1) retain this form for any independent variable.

In order that a second set of functions $\overline{\Gamma}^i_{jk}$ define the same paths, we have on subtracting (1.1) from similar equations in the $\overline{\Gamma}$'s,

$$A^{ij}_{hkl}\frac{dx^h}{dt}\frac{dx^k}{dt}\frac{dx^l}{dt} = 0,\ A^{ij}_{hkl} = \delta^i_h(\overline{\Gamma}^j_{kl} - \Gamma^j_{kl}) - \delta^j_h(\overline{\Gamma}^i_{kl} - \Gamma^i_{kl}).$$

Since these equations must be satisfied for every path, we must have

$$A^{ij}_{hkl} + A^{ij}_{klh} + A^{ij}_{lhk} = 0. \quad (1.2)$$

Contracting these equations for i and h, we have

$$\Gamma^i_{kl} - \frac{1}{n+1}(\delta^i_k\Gamma^h_{hl} + \delta^i_l\Gamma^h_{hk}) = \overline{\Gamma}^i_{kl} - \frac{1}{n+1}(\delta^i_k\overline{\Gamma}^h_{hl} + \delta^i_l\overline{\Gamma}^h_{hk}). \quad (1.3)$$

When these conditions are satisfied, so also are equations (1.2). If we define functions π^i_{kl} by

$$\pi^i_{kl} = \Gamma^i_{kl} - \frac{1}{n+1}(\delta^i_k\Gamma^h_{hl} + \delta^i_l\Gamma^h_{hk}), \quad (1.4)$$

then the equations of the paths may be written

$$\frac{dx^i}{dt}\left(\frac{d^2x^j}{dt^2} + \pi^j_{kl}\frac{dx^k}{dt}\frac{dx^l}{dt}\right) - \frac{dx^j}{dt}\left(\frac{d^2x^i}{dt^2} + \pi^i_{kl}\frac{dx^k}{dt}\frac{dx^l}{dt}\right) = 0. \quad (1.1')$$

From (1.4) we have

$$\pi^i_{il} = 0, \quad (1.5)$$

and from (1.3) we obtain $\pi^i_{jk} = \overline{\pi}^i_{jk}$. The functions π^i_{jk}, which accordingly are independent of the choice of the Γ's subject to the relations (1.3), we call the *projective coefficients* of connection, following T. Y. Thomas.[1]

When the coördinates x^i are subjected to a general analytic transformation into coördinates x'^i and the equations of the paths in the new coördinates are written

$$\frac{dx'^i}{dt}\left(\frac{d^2x'^j}{dt^2} + \Gamma'^j_{kl}\frac{dx'^k}{dt}\frac{dx'^l}{dt}\right) - \frac{dx'^j}{dt}\left(\frac{d^2x'^i}{dt^2} + \Gamma'^i_{kl}\frac{dx'^k}{dt}\frac{dx'^l}{dt}\right) = 0,$$

it can be shown[3] that the Γ's and Γ''s are related by the equations

$$\frac{\partial^2 x'^{\alpha}}{\partial x^i \partial x^j} = \pi^h_{ij} \frac{\partial x'^{\alpha}}{\partial x^h} - \pi'^{\alpha}_{\beta\gamma} \frac{\partial x'^{\beta}}{\partial x^i} \frac{\partial x'^{\gamma}}{\partial x^j} + \frac{\partial x'^{\alpha}}{\partial x^i} \frac{\partial \theta}{\partial x^j} + \frac{\partial x'^{\alpha}}{\partial x^j} \frac{\partial \theta}{\partial x^i}, \quad (1.6)$$

where $\pi'^{\alpha}_{\beta\gamma}$ are defined in terms of $\Gamma'^{\alpha}_{\beta\gamma}$ by equations of the form (1.4), and where we have put

$$\theta = \frac{1}{n+1} \log \Delta, \quad \Delta = \left| \frac{\partial x'}{\partial x} \right|, \quad (1.7)$$

that is, Δ is the jacobian of the transformation.

For a coördinate system x^i a parameter x can be chosen for each path so that the equations of the path are

$$\frac{d^2 x^i}{dx^2} + \pi^i_{jk} \frac{dx^j}{dx} \frac{dx^k}{dx} = 0. \quad (1.8)$$

We call x the *projective parameter* of the path for the x's, following T. Y. Thomas.[1] By means of (1.6) we show that the projective parameter x' for the same path in the coördinate system x'^i is given by

$$x' = \int_0^x e^{2\theta} dx. \quad (1.9)$$

We consider all the paths through a point P of coördinates x^i_0 and without loss of generality we choose the parameter x so that $x = 0$ at P. If we assign coördinates y^i at points of each path through P by means of the equations

$$y^i = \left(\frac{dx^i}{dx}\right)_0 x, \quad (1.10)$$

we find,[4] on expanding the solution of (1.18) as power series, that

$$x^i = x^i_0 + y^i - \frac{1}{2} (\pi^i_{jk})_0 y^j y^k \ldots - \frac{1}{r} (\pi^i_{j_1 \ldots j_r})_0 y^{j_1} \ldots y^{j_r} + \ldots, \quad (1.11)$$

where the functions $\pi^i_{j_1 \ldots j_r}$ are determinate functions of π^i_{jk} and their derivatives of order $r - 2$ at most, and where $(\ )_0$ indicates the value at P.

Thus equations (1.11) are the equations of transformation of coördinates x^i and y^i. These equations obtain within a domain about P such that no two paths through P, which lie entirely within the domain, meet again in the domain.

The transformation (1.11) belongs to the class of transformations of coördinates related to the x's by equations of the form

$$x^i = x^i_0 + y^i - \frac{1}{2} (\pi^i_{jk})_0 y^j y^k + \varphi^i, \quad (1.12)$$

where φ^i are functions of the y's which with their first and second derivatives vanish when the y's are zero. We have called any set of y's so defined *projective coördinates,*[5] and have shown that because of (1.5) we have

$$\left(\frac{\partial}{\partial y^i} \log \Delta\right)_0 = 0, \tag{1.13}$$

where Δ is the jacobian $\left|\frac{\partial x}{\partial y}\right|$.

From (1.10) we have

$$\frac{d^2 y^i}{dx^2} = 0, \frac{dy^i}{dx} = \frac{y^i}{x}. \tag{1.14}$$

If we denote by $\bar{\pi}^i_{jk}$ the projective coefficients in the y's and remark that x is not the projective parameter for the y's, it follows from equations of the form (1.1′) in the y's and from (1.14) that

$$\frac{\bar{\pi}^i_{jk} y^j y^k}{y^i} = \frac{\bar{\pi}^l_{jk} y^j y^k}{y^l}. \tag{1.15}$$

Expressing the x's as functions of the y's, we have that equations (1.8) reduce to

$$\left(\frac{\partial^2 x^i}{\partial y^j \partial y^k} + \pi^i_{hl} \frac{\partial x^h}{\partial y^j} \frac{\partial x^l}{\partial y^k}\right) y^j y^k = 0,$$

which because of equations of the form (1.6) are reducible to

$$\left(\bar{\pi}^h_{jk} y^j y^k + 2\, y^h \frac{\partial \theta}{\partial y^j} y^j\right) \frac{\partial x^i}{\partial y^h} = 0,$$

where θ is given by (1.7) with $\Delta = \left|\frac{\partial x}{\partial y}\right|$. Since Δ is not equal to zero, these equations are equivalent to

$$\bar{\pi}^h_{jk} y^j y^k = -2 y^h \frac{\partial \theta}{\partial y^j} y^j. \tag{1.16}$$

2. If φ is any function of the y's and we put

$$z^i = \frac{y^i}{\varphi}, \quad \psi = \varphi - \frac{\partial \varphi}{\partial y^i} y^i, \tag{2.1}$$

we have

$$\frac{\partial z^i}{\partial y^j} y^j = \frac{y^i}{\varphi^2} \psi, \quad \frac{\partial \psi}{\partial y^j} y^j = -\frac{\partial^2 \varphi}{\partial y^i \partial y^j} y^i y^j, \tag{2.2}$$

$$\frac{\partial^2 z^i}{\partial y^j \partial y^k} y^j y^k = \frac{y^i}{\varphi^3}\left(\varphi \frac{\partial \psi}{\partial y^j} y^j - 2\psi \frac{\partial \varphi}{\partial y^k} y^k\right). \tag{2.3}$$

If $\bar{\Delta}$ denotes the jacobian $\left|\frac{\partial z}{\partial y}\right|$, we find that[6]

$$\bar{\Delta} = \frac{\psi}{\varphi^{n+1}}, \quad \bar{\theta} = \frac{1}{n+1}\log\frac{\psi}{\varphi^{n+1}}. \tag{2.4}$$

Denoting by P^i_{jk} the projective coefficients in the z's, we have from (2.3), from equations of the form (1.6) in the z's and y's, and from (1.16)

$$y^i y^j \frac{\partial}{\partial y^j}\log(\psi^{n-1}\Delta^2) + (n+1)\, P^i_{jk} z^j z^k . \psi = 0,$$

where $\Delta = \left|\frac{\partial x}{\partial y}\right|$. Hence, if we choose the function φ so that

$$\varphi - \frac{\partial \varphi}{\partial y^j} y^j = \Delta^{\frac{2}{1-n}}, \tag{2.5}$$

we have for the coördinates z^i the equations

$$P^i_{jk} z^j z^k = 0. \tag{2.6}$$

Because of (1.5) it follows from (1.11) that Δ is of the form

$$\Delta = 1 + a_{ij} y^i y^j + \ldots$$

Consequently, $\Delta^{\frac{2}{1-n}}$ is of the form

$$\Delta^{\frac{2}{1-n}} = 1 + b_{ij} y^i y^j + \ldots + b_{i_1 \ldots i_r} y^{i_1}_1 \ldots y^{i_r} + \ldots,$$

and this expansion holds for the domain about P within which $\Delta \neq 0$. A solution of (2.5) is given by

$$\bar{\varphi} = 1 - \left(b_{ij} y^i y^j + \ldots + \frac{1}{r-1} b_{i_1 \ldots i_r} y^{i_1} \ldots y^{i_r} + \ldots\right). \tag{2.7}$$

Making use of (1.10) we see that along each path through P the domain of convergence of the right-hand member of (2.7) is the same as for the above expression for $\Delta^{\frac{2}{1-n}}$. Consequently these two expansions have the same domain.

From the form of equation (2.5) it follows that the general solution of this equation is

$$\varphi = \bar{\varphi} + \tau, \tag{2.8}$$

where τ is an arbitrary homogeneous function of the first degree in the y's. At P we have $(\varphi)_0 = 1$ whatever be τ. Then from (1.11) and (2.1) we have

$$\left(\frac{\partial x^i}{\partial y^j}\right)_0 = \left(\frac{\partial z^i}{\partial y^j}\right)_0 = \left(\frac{\partial y^i}{\partial z^j}\right)_0 = \left(\frac{\partial x^i}{\partial z^j}\right)_0 = \delta^i_j. \tag{2.9}$$

If any solution φ of (2.5) is used in (2.1) to define coördinates z^i and we define a parameter z for each path by

$$z = \frac{x}{\varphi}, \tag{2.10}$$

we have, in consequence of (1.14) and (2.2),

$$\frac{dz}{dx} = \frac{\psi}{\varphi^2}, \quad \frac{dz^i}{dz} = \frac{dy^i}{dx}, \tag{2.11}$$

and because of (1.14)

$$\frac{d^2 z^i}{dz^2} = 0. \tag{2.12}$$

In view of this result and (2.6) we have that z defined by (2.10) is the projective parameter for the coördinates z^i. Since $(\varphi)_0 = 1$ and $(\psi)_0 = 1$, as follows from (2.5), we have from (2.11) that $\left(\frac{dz}{dx}\right)_0 = 1$. Consequently we obtain from (1.10), (2.1) and (2.10)

$$z^i = \left(\frac{dx^i}{dz}\right)_0 z. \tag{2.13}$$

If for any solution φ of (2.5) equations (2.1) are solved for the y's as functions of the z's and these expressions are substituted in (1.11), the resulting equations give the relations between the x's and the z's. In order that the latter be projective coördinates, it is necessary that

$$\left(\frac{\partial^2 x^i}{\partial y^j \partial y^k}\right)_0 = \left(\frac{\partial^2 x^i}{\partial z^j \partial z^k}\right)_0,$$

from which in consequence of (2.9) it follows that the solution φ must be such that

$$\left(\frac{\partial^2 z^i}{\partial y^j \partial y^k}\right)_0 = 0.$$

Since $(\varphi)_0 = 1$, we have from (2.1) that we must have $\left(\frac{\partial \varphi}{\partial y^i}\right)_0 = 0.$

This condition is satisfied by $\bar{\varphi}$, defined by (2.7), and this is the only solution, because τ in (2.8) being any homogeneous function of first degree in the y's does not satisfy the condition $\left(\frac{\partial \tau}{\partial y^i}\right)_0 = 0$. Accordingly, if in (2.1) we use the function $\bar{\varphi}$, defined by (2.7), the coördinates z^i are such that

$$\left(\frac{\partial}{\partial z^j} \log \left|\frac{\partial x}{\partial z}\right|\right)_0 = 0, \tag{2.14}$$

as follows from (1.14). We call the z's, thus defined uniquely with respect to the x's, the *projective normal coördinates* of the space corresponding to the x's.

3. If we take another general coördinate system x'^i and define projective coördinates z'^i, with origin at P by means of the function $\bar{\varphi}'$ analogous to $\bar{\varphi}$ defined by (2.7), then in consequence of (2.9) and (2.13) we have

$$z'^i = \left(\frac{dx'^i}{dz'}\right) z' = \left(\frac{\partial x'^i}{\partial x^j}\frac{dx^j}{dz}\frac{dz}{dz'}\right)_0 z' = a^i_j z^j \left(\frac{dz}{dz'}\right)_0 \frac{z'}{z},$$

where the constants a^i_j are defined by

$$\left(\frac{\partial x'^i}{\partial x^j}\right)_0 = a^i_j. \tag{3.1}$$

Between the projective parameters z and z' for the respective coördinate systems z^i and z'^i and the jacobian of the transformation of these coördinates the following relation holds:[7]

$$\frac{dz'}{dz} = \left|\frac{\partial z'}{\partial z}\right|^{\frac{2}{n+1}}. \tag{3.2}$$

In consequence of (2.9)

$$\left(\left|\frac{\partial z'}{\partial z}\right|\right)_0 = \left(\left|\frac{\partial x'}{\partial x}\right|\right)_0 = |a^i_j| \equiv A.$$

Consequently we have

$$z'^i = a^i_j z^j \frac{z'}{z} A^{-\frac{2}{n+1}}.$$

Since the relation between the z's and z''s is independent of the paths, we must have

$$\frac{z'}{z} = \frac{A^{\frac{2}{n+1}}}{f} \tag{3.3}$$

where f is a determinate function of the z's. Differentiating this equation we have

$$\frac{dz'}{dz} = \frac{1}{f^2}\left(f - \frac{\partial f}{\partial z^j} z^j\right) A^{\frac{2}{n+1}}. \tag{3.4}$$

From the above equations follow the equations

$$z'^i = \frac{a^i_j z^j}{f}. \tag{3.5}$$

The jacobian of this transformation is[8]

$$\left|\frac{\partial z'}{\partial z}\right| = \frac{1}{f^{n+1}}\left(f - \frac{\partial f}{\partial z^j} z^j\right) A. \tag{3.6}$$

Then from (3.2), (3.4) and (3.6) we have

$$f - \frac{\partial f}{\partial z^j} z^j = 1,$$

of which the general solution is $1 + \sigma$, where σ is a homogeneous function of the first degree in the coördinates z^i. Consequently (3.6) reduces to

$$\left|\frac{\partial z'}{\partial z}\right| = \frac{1}{(1+\sigma)^{n+1}} A. \tag{3.7}$$

In consequence of (2.14), (2.9) and similar equations in the x''s and z''s, of (3.7), and the law of multiplication of jacobians we have

$$\left(\frac{\partial}{\partial x^i} \log \left|\frac{\partial x'}{\partial x}\right|\right)_0 = \left(\frac{\partial}{\partial z^i} \log \left|\frac{\partial z'}{\partial z}\right|\right)_0 = -(n+1)\left(\frac{\partial \sigma}{\partial z^i}\right)_0.$$

If then from the transformation of the x's and x''s we have constants a_i defined by

$$\left(\frac{\partial}{\partial x^i} \log \left|\frac{\partial x'}{\partial x}\right|\right)_0 = -a_i(n+1), \tag{3.8}$$

we obtain $\left(\frac{\partial \sigma}{\partial z^i}\right)_0 = a_i$. Put $\sigma = a_i z + \rho$, where ρ is a homogeneous function of the first degree. Since we must have $\left(\frac{\partial \rho}{\partial z^i}\right)_0 = 0$, it follows that $\rho = 0$. Consequently the relation between the z's and z''s is the linear fractional form

$$z'^i = \frac{a^i_j z^j}{1 + a_j z^j}, \tag{3.9}$$

where a^i_j and a_i are defined by (3.1) and (3.8), respectively.

4. In §2 it was seen that by taking for φ any solution of equation (2.5) the coördinates z^i defined by (2.1) satisfy (2.6), (2.12) and (2.13). We shall show that there is a solution φ' of the equation in the y''s analogous to (2.5) for which the coördinates z'^i are given by (3.9).

If we proceed with the equations $y'^i = \left(\frac{dx'^i}{dx'}\right)_0 x'$ in a manner similar to that which led to (3.3), (3.4), (3.5) and (3.6), we get

$$\frac{x'}{x} = \frac{A^{\frac{2}{n+1}}}{\rho}, \quad \frac{dx'}{dx} = \frac{1}{\rho^2}\left(\rho - \frac{\partial \rho}{\partial y^i} y^i\right) A^{\frac{2}{n+1}},$$
$$y'^i = \frac{a^i_j y^j}{\rho}, \quad \left|\frac{\partial y'}{\partial y}\right| = \frac{1}{\rho^{n+1}}\left(\rho - \frac{\partial \rho}{\partial y^i} y^i\right) A, \tag{4.1}$$

where ρ is a function of the y's. Moreover, since x and x' are the projective parameters for the x's and x''s, we have[7]

$$\frac{dx'}{dx} = \left|\frac{\partial x'}{\partial x}\right|^{\frac{2}{n+1}}. \tag{4.2}$$

If we put

$$\varphi' = \frac{\varphi + a_j y^j}{a^i_j y^j} y'^i \qquad (i \text{ not summed}), \tag{4.3}$$

where a_j are any constants, because of (4.1) we have

$$\varphi = -a_j y^j + \rho\varphi'.$$

In consequence of (4.1) we have

$$\frac{\partial \varphi}{\partial y^i} y^i = -a^i y^i + \frac{\partial \rho}{\partial y^i} \cdot y^i \cdot \varphi' + \frac{\partial \varphi'}{\partial y'^k} \frac{\partial y'^k}{\partial y^i} y^i$$
$$= \varphi - \left(\rho - \frac{\partial \rho}{\partial y^i} y^i\right)\left(\varphi' - \frac{\partial \varphi'}{\partial y'^j} y'^j\right). \tag{4.4}$$

From (4.1) and (4.2) we have

$$\left|\frac{\partial x'}{\partial x}\right| = \frac{1}{\rho^{n+1}} A \left(\rho - \frac{\partial \rho}{\partial y^i} y^i\right), \qquad \left|\frac{\partial x'}{\partial x}\right| \cdot \left|\frac{\partial y}{\partial y'}\right| = \left(\rho - \frac{\partial \rho}{\partial y^i} y^i\right)^{\frac{n-1}{2}}.$$

Hence from (4.4) and (2.5) we have

$$\varphi' - \frac{\partial \varphi'}{\partial y'^j} y'^j = \left|\frac{\partial x'}{\partial y'}\right|^{\frac{2}{n-1}},$$

which is the analogous equation in the y''s. Using φ' defined by (4.3) in the definition of the z'^i, we have the equations (3.9). Thus we have general sets of coördinates satisfying (3.9) where a_j are arbitrary constants. But these are projective normal coördinates only when $\bar{\varphi}$, defined by (2.7), and the analogous $\bar{\varphi}'$ are used for the definition of z^i and z'^i, and the constants a_j are given by (3.8).

[1] "On the Projective and Equiprojective Geometries of Paths," *Proc. Nat. Acad. Sci.*, **11**, pp. 199–203, 1925.

[2] "Projective Normal Coördinates for the Geometry of Paths," *Ibid.*, **11**, pp. 204–207, 1925.

[3] "Non-Riemannian Geometry," *Am. Math. Soc. Colloq. Publica.*, **8**, 1927. Hereafter a reference to this book is of the form E §45.

[4] E. §§22, 40. Formula (22.8) is not correct so as to insure that the quantities are symmetric in all the indices; the form to insure this requirement is readily obtained.

[5] E., p. 111.

[6] E., §41.

[7] E., p. 107.

[8] E., p. 113.

ON THE NORM-RESIDUE SYMBOL IN THE THEORY OF CYCLOTOMIC FIELDS

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Let l be an odd prime and $\zeta = e^{2i\pi/l}$; $\mathfrak{l} = (1 - \zeta)$; k the field defined by ζ and ω and θ integers in k prime to $\mathfrak{l}$. Then one way of expressing the norm residue symbol for the field k is

$$\left\{\frac{\omega,\ \theta}{\mathfrak{l}}\right\} = \zeta^{P},$$

where

$$P \equiv -l_1(\theta)\,\frac{N(\omega) - 1}{l} + l_1(\omega)\,\frac{N(\theta) - 1}{l} + \sum_{s=2}^{l-2} (-1)^{s-1}\, l_s(\theta)\, l_{l-s}(\omega) \pmod{l}; \qquad (1)$$

$$\theta = a_0 + a_1\,\zeta + \ldots + a_{l-2}\,\zeta^{l-2},$$

the a's rational integers;

$$\theta(e^v) = a_0 + a_1\,e^v + a_2\,e^{2v} + \ldots + a_{l-2}\,e^{(l-2)v};$$

$$l_s(\theta) = \left[\frac{d^s \log \theta(e^v)}{dv^s}\right]_{v=0};$$

$N(\theta)$ is the norm of θ; with similar definitions for ω. This symbol has been treated by a number of writers.[1] So far, apparently, the value of P has always been represented by summation of the type mentioned above or analogous to it. In the present note I shall give a method for carrying out this summation. To effect this we expand the expression